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Name Khaled Jadallah, ID 38264

Exam II , MTH 221 , Spring 2011

Ayman Badawi

QUESTION 1. (POINTS) For each question below, circle the right answer.

(i) One of the following is a subspace of R^3:

a. $S = \{(a+b, -2a, b+1) \mid a, b \in R\}$
b. $S = \{(a-2b, b^2 - a, 0) \mid a, b \in R\}$
c. $S = \{(3a, 2b+a, 5) \mid a, b \in R\}$
d. $S = \{(0, 0, 3a+2b) \mid a, b \in R\}$

(ii) One of the following is a subspace of P_2:

a. $S = \{(a+b) + 2x \mid a, b \in R\}$
b. $S = \{a + bx + (a-b)x^2 \mid a, b \in R\}$
c. $S = \{3 + ax \mid a \in R\}$
d. $S = \{a + 3ax \mid a \in R\}$

(iii) One of the following is a linear transformation from R^3 to R^3:

a. $T : R^3 \to R^3, T(a,b,c) = (0, 1, a+b+c)$
b. $T : R^3 \to R^3, T(a,b,c) = (3a + b^2, c, 0)$
c. $T : R^3 \to R^3, T(a,b,c) = (a+b, 0, c-2b)$
d. $T : R^3 \to R^3, T(a,b,c) = (3a, -b, 2+c)$

(iv) Let A be a 4×3 matrix such that the homogenous system $AX = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$ has a unique solution. One of the following statements is correct:

a. The rows of A are independents.
b. Rank(A) is at most 2.
c. The columns of A are independent
d. Exactly one column of A is a linear combination of the other two columns.

(v) If $T : R^5 \to R^3$ is a linear transformation such that $dim(Ker(T)) = 3$, then one of the following statement is correct:

a. The range of T equals to R^3.
b. The range of T "lives" inside R^3 but not equal to R^3
c. It is possible to find three independent points in the range of T.
d. If A, B are independent points in R^3, then Range of T = $Span\{A, B\}$.
e. (a) and (c)

(vi) Let $T : P_2 \to R$ be a linear transformation such that $T(2+x) = 4, T(-1-x) = -4$. Then $T(2) =$

a. 2.
b. 0.
c. -2
d. None of the above.

(vii) Given $S = \begin{bmatrix} a+b & 0 & -a-b \\ c & 0 & -c \end{bmatrix} \mid a, b, c \in R\}$ is a subspace of $R^{2\times 3}$. Then $dim(S) =$

a. 2
b. 3
c. 6
d. None of the above.

(viii) Let $S = span\{(1,-1,0), (2,-1,0), (1,0,0)\}$. Then $dim(S) =$

a. 2
b. 3
c. 1
d. None of the above

⇒ (ix) Let $S = span\{(1,1,0), (1,-1,1), (0,-2,1)\}$. Then

a. Every point in R^3 belongs to S.
b. dim(S) = 3
c. (a) and (b) are correct
d. The point (4, -2, 0) does not belong to S.
~~e.~~ None of of the above

(x) Let S be a subspace of R^4 such that $dim(S) = 3$. One of the following statements is correct:

a. Every 3 points in S form a basis for S.
b. There are exactly 3 independent points in S.
~~c.~~ It is possible that the span of 4 independent points in R^4 equals to S
d. None of the above

[Correct answer (d), but if you circle (B), then ok]

(xi) One of the following is a basis for P_3:

a. $B = \{1 + x^2, -x + x^2, 3, x\}$
b. $B = \{1 + x, 2, -2x\}$
c. $B = \{1 + x + x^2, x + x^2, 1 + x^2\}$
d. $B = \{1 + x, 3 + x + x^2, 2 + x^2\}$

⇒ (xii) Let A be a 4×4 matrix such that $det(A) = 3$. Then one of the following statements is correct:

a. Rank(A) is at most 3. ✗
b. span of the columns of A "lives" in R^4 but not equal R^4. ✗
c. Rank(A) = 4.
d. At least one column of A is a linear combination of the other three columns. ✗

(xiii) Let $D = \{a + (b+a)x + 3bx^2 \mid a, b \in R\}$ be a subspace of P_3. Then a basis for D is :

a. $B = \{1 + x, x^2\}$
b. $B = \{1 + x, x + 3x^2\}$
c. $B = \{x + 3x^2, 1 + 2x\}$
d. All the above.

(xiv) Let $T : R^2 \to R$ be a linear transformation such that $T(1,0) = 4, T(2,-2) = 2$. Then the standard matrix representation of T is :

a. $\begin{bmatrix} 4 & 2 \end{bmatrix}$
b. $\begin{bmatrix} 4 & 3 \end{bmatrix}$
c. $\begin{bmatrix} 4 & -2 \end{bmatrix}$
d. None of the above

(xv) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 0 \\ 2 & 2 & 2 & 2 & -2 \end{bmatrix}$. Then $N(A) =$

a. span$\{(-1,1,0,0,0),(-1,0,1,0,0),(-1,0,0,1,0)\}$
b. span$\{(-1,1,1,1,0)\}$
c. span$\{(1,-1,-1,0,0),(1,0,0,-1,0),(-1,0,0,0,1)\}$
d. span $\{(1,0,0,0,0),(0,1,0,0,0),(0,0,1,0,0)\}$
e. None of the above

(xvi) Let A be the above matrix. Then Column(A) =

a. Span$\{(1,-1,2),(1,0,-2)\}$
b. Span$\{(1,0,0),(1,1,0)\}$
c. Span$\{(1,0,0),(1,1,-4)\}$
d. All the above are correct.

(xvii) Let $T : P_3 \to R$ such that $T(a+bx+cx^2) = \int_0^1 (a+bx+cx^2)\,dx$. Then $ker(T) =$

a. Span$\{1, x, x^2\}$
b. Span$\{-0.5+x\}$
c. Span $\{\frac{-2.5}{3} + x + x^2\}$
d. Span $\{-0.5+x, \frac{-1}{3} + x^2\}$
e. None of the above

(xviii) Let $T = R^4 \to R^3$ such that $T(a,b,c,d) = (a+b+c-2d, -2d, d)$. We know T is a linear transformation. The standard matrix representation of T is

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ -2 & -2 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 1 & 1 & -2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(xix) In the Previous Question, $dim(Ker(T)) =$

a. 1
b. 3
c. 2
d. 4

(xx) In Question number 18 (XVIII). $range(T) =$

a. Span $\{(1,1,1,-2),(0,0,0,-2)\}$
b. Span $\{(1,0,0,-2)\}$
c. Span $\{(1,0,0),(-2,-2,1)\}$
d. Span $((1,0,0),(0,-2,0))$
e. None of the above

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab